Value at Risk
Ch.9

Related Learning Objectives

2b) Evaluate how risks are correlated, and give examples of risks that are positively correlated and risks that are negatively correlated

2c) Analyze and evaluate risk aggregation techniques, including use of correlation, integrated risk distributions and copulas

3c) Analyze quantitative financial data and construct measures from insurance data using modern statistical methods (including asset prices, credit spreads and defaults, interest rates, incidence, causes and losses). Contrast the available range of methods with respect to scope, coverage and application

Key Points of This Reading

1) Understand the three ways to model volatility.

2) Understand the implied volatility (or implied standard deviation)

Volatility

Volatility Prediction is Important for Risk Management

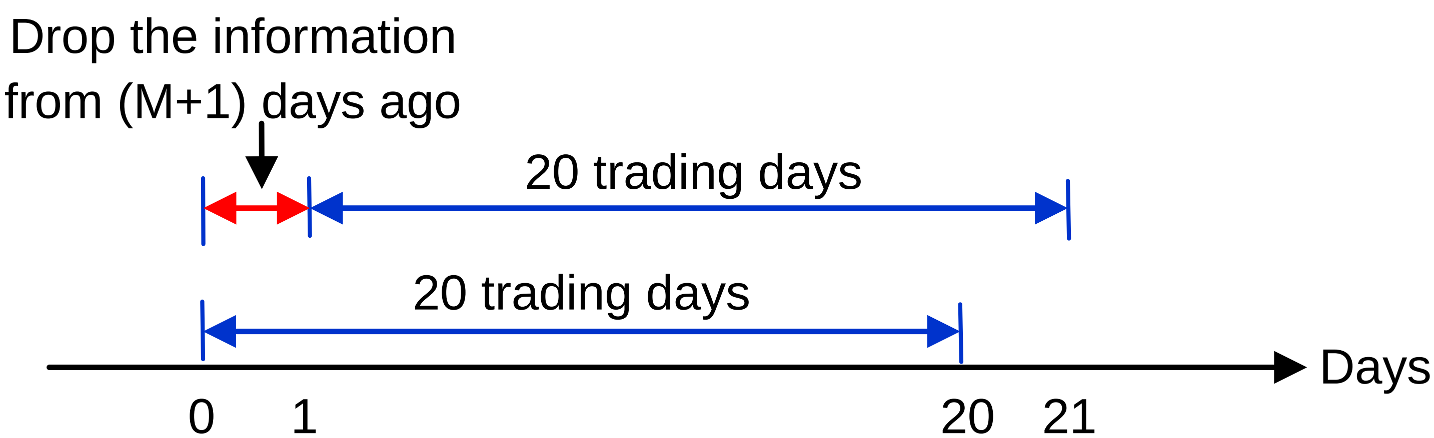
1. Measure volatility
2. Predict impact
3. Control risk

Three Ways to Model Volatility

1. Moving Average
2. GARCH Estimation
3. EWMA Estimation (RiskMetrics)

#1. Moving Average (MA)

Volatility estimate = $\sigma_t^2 = \dfrac{1}{M}\sum_{i=1}^{M} r_{t-1}^2$



Drawbacks of this Method

1. It ignores the dynamic ordering of observations
2. The volatility estimate may drop without reason

#2. GARCH

GARCH(1,1) Process: $h_t = \alpha_0 + \alpha_1 r_{t-1}^2 + \beta h_{t-1}$ where $\alpha_1 + \beta < 1$

Advantages	Disadvantages
1. It offers a model with few parameters that fits the data well	1. The drawback of GARCH models is their nonlinearity
2. GARCH models can be used in the markets that display volatility clustering	2. The parameters must be estimated by maximization of the likelihood function
3. Many variants of the GARCH model are available	

#2. GARCH

GARCH(1,1) Process: $h_t = \alpha_0 + \alpha_1 r_{t-1}^2 + \beta h_{t-1}$ where $\alpha_1 + \beta < 1$

Example

$h_t = 0.000002 + 0.13 r_{t-1}^2 + 0.86 h_{t-1}$ / $h_{t-1} = 0.016^2$ / $r_{t-1}^2 = 0.01^2$

Calculate the new estimate of the volatility

$$h_t = 0.000002 + 0.13 r_{t-1}^2 + 0.86 h_{t-1}$$
$$= 0.000002 + 0.13(0.01^2) + 0.86(0.016^2)$$
$$= 0.00023516$$

The new estimate of the volatility

$= \sqrt{0.00023516} = 0.0153 = 1.53\%$

#3. EWMA (RiskMetrics)

EWMA Process: $h_t = \lambda h_{t-1} + (1-\lambda) r_{t-1}^2$

Advantages	Disadvantages
1. The exponential model is particularly easy to implement because it relies on one parameter only	1. It is difficult to estimate the parameter daily
2. The estimator is recursive	2. The decay factor may vary not only across series but also over time, thus losing consistency over different periods
	3. Different values of λ create incompatibilities across the covariance terms and may lead to unreasonable values for correlations
	4. The model does not allow mean reversion

#3. EWMA (RiskMetrics)

This EWMA model is a special case of the GARCH process

GARCH: $h_t = \alpha_0 + \alpha_1 r_{t-1}^2 + \beta h_{t-1}$

$$h_t = 0 + (1-\lambda) r_{t-1}^2 + \lambda h_{t-1} \quad (\alpha_0 = 0, \alpha_1 = 1-\lambda, \beta = \lambda)$$

$h_t = \lambda h_{t-1} + (1-\lambda) r_{t-1}^2$ ← **EWMA model**

Modeling Correlations

MA, GARCH, and EWMA can be used to capture time variation in correlation

Correlation: $$\rho_{12,t} = \frac{h_{12,t-1}}{\sqrt{h_{1,t-1}h_{2,t-1}}}$$